# Stochastic Muscle Results

This document describes the problem that is solved to show that muscle co-contraction is optimal for some tasks. The idea is that co-contraction is optimal because it would cost more energy to use feedback to control the system than to co-contract the muscles, which makes the joint stiffer. This problem is analyzed on an arm-like pendulum.

## Problem

The goal of this problem is to keep and arm-like pendulum upright in a stochastic environment, while minimizing muscle activation. Therefore, there are two objectives that should be minimized, the muscle activation and the deviation from the upright position (at pi/2). A weighting between the objectives defines how much emphasis is put on each. This weight is applied to the deviation part.

This optimization problem is solved with direct collocation. This means that one-step dynamics are added to the constraint. Also, a task constraint is added. This task constraint ensures the periodicity of the motion, such that theoretically, this task could be performed forever.

The control and dynamics were set up as follows. A separate controller was used for each muscle. Each has one open loop control input, a position feedback and a derivative feedback. So there are six controller parameters. The length of the simulation was chosen randomly, 10 seconds, with 600 time nodes. For each of these nodes, there are six states, the angle, the angular velocity, two muscle activation states and two fiber lengths (ratio).

## Solution Method

Results were obtained for different weightings in the objective and for different noise standard deviations. Because this problem is stochastic, each problem was repeated ten times to make sure the solution is not specific for the noise that is used. For the increasing standard deviation, the same random values were used as for the previous result with a smaller standard deviation, while using that solution as initial guess with a warm start in IPOPT. This enabled us to get the results fast.

## Results

The first figure on the left shows the co-contraction input as a function of the standard deviation for the largest weight that was used. With 0 standard deviation, the problem is deterministic, so there is no co-contraction required and there is also no variation among the solutions. After that, the amount of co-contraction increases with the standard deviation and so does the variation.

The right figure shows the co-contraction input as a function of the weights for the highest standard deviation. Note that a logarithmic scale is used on the x-axis. For the lowest weight, there is hardly any co-contraction, because to minimize effort only would be to just let the pendulum move as much as the noise lets it while only using some control to adjust for the periodicity constraint. The amount of co-contraction increases with the weight, which is as expected. This is almost linear or maybe quadratic on a logarithmic scale.



The figures below show the position feedback gain as a function of standard deviation and weight, for the same results as above. The derivate feedback gain was not added because it was equal to (close to) zero for all solutions. The position feedback gains are the same in both muscles in the optimal solution, because no equality constraints were used. Note that one gain is negative and the other is positive due to the direction in which the muscles operate (opposite of each other).

It is interesting to see that the position feedback gain is the same for the different standard deviations, which means that the controller is independent on the amount of noise. The feedback gain increases with the weight of the deviation objective. This makes sense because the higher the weight, the less deviation is desired.



The following table shows the mean RMS error between the pendulum angle and the upright position. It increases with the standard deviation and decreases with the objective weight, which makes sense. The results are in degrees. The figure shows the same thing. It shows that the increase is linear with the standard deviation, while it is more quadratic with the logarithmic weight scale.

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| --- | --- | --- | --- | --- | --- |
| StDev/Weight | 1e-1 | 1 | 10 | 100 | 1000 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.2 | 0.7473 | 0.6377 | 0.3247 | 0.1289 | 0.0514 |
| 0.4 | 1.4947 | 1.2592 | 0.5989 | 0.2332 | 0.0932 |
| 0.6 | 2.2422 | 1.8631 | 0.8452 | 0.3278 | 0.1325 |
| 0.8 | 2.9899 | 2.4494 | 1.0770 | 0.4194 | 0.1716 |
| 1 | 3.7382 | 3.0186 | 1.3013 | 0.5110 | 0.2110 |



The final table shows the relative standard deviation of the objective function. This means that the standard deviation of the objective was divided by the mean value of that objective. This table is used to see how much variation there is among the optimal solutions with different random values. The standard deviation decreases with increasing weight, because the problem becomes more quadratic and less dependent on the muscle dynamics. Except for the solution with very low weight, the deviation is less than 30% of the mean value, which I think is reasonable.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| StDev/Weight | 1e-1 | 1 | 10 | 100 | 1000 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.2 | 0.4786 | 0.2692 | 0.2609 | 0.1295 | 0.1293 |
| 0.4 | 0.4787 | 0.2672 | 0.2451 | 0.1251 | 0.1308 |
| 0.6 | 0.4788 | 0.2650 | 0.2354 | 0.1246 | 0.1343 |
| 0.8 | 0.4789 | 0.2628 | 0.2298 | 0.1255 | 0.1377 |
| 1 | 0.4792 | 0.2606 | 0.2266 | 0.1269 | 0.1407 |